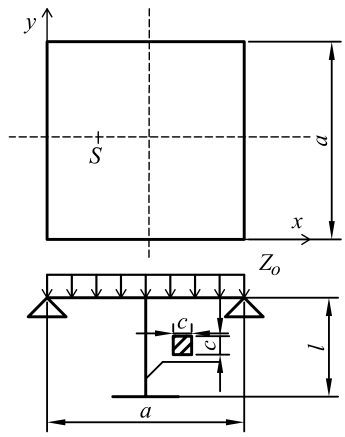
Primer 4

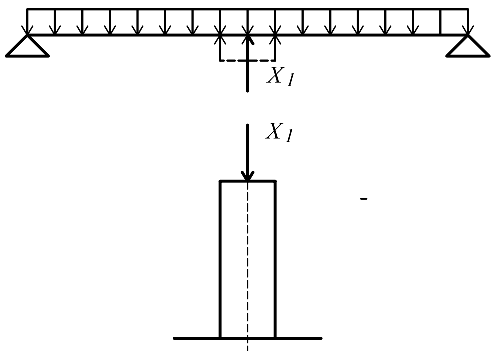
Za ploču prikazanu na slici odrediti ugib i presečne sile u tački S.

$w_s, T_{x,s}, M_{xs}, M_{xy,s} = ?$

$Z(x,y) = Z_0 = const.$



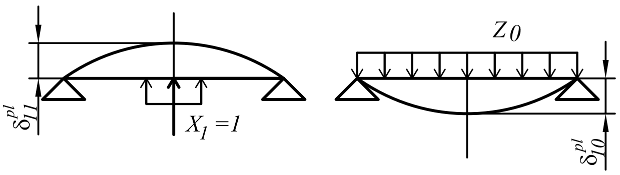
Rešenje:



Uslovna jednačina: $\delta_{11} X_1 + \delta_{10} = 0$.

$\delta_{11} = \delta_{11}^P + \delta_{11}^S$

$\delta_{10} = \delta_{10}^P$



$$\delta_{11}^{PL} = w^{X_1}\left(x = \frac{a}{2}, y = \frac{a}{2}\right)$$

$$\delta_{11}^{S} = \Delta l^{X_1} = \frac{X_1}{EF} \cdot l = \frac{l}{EF} = \frac{l}{E \cdot c^2}$$

$$\delta_{10}^{P} = w^{Z_0}\left(x = \frac{a}{2}, y = \frac{a}{2}\right)$$

$$X_1 = -\frac{\delta_{10}}{\delta_{11}}$$

$$w_S = w_0 + w_1 \cdot X_1$$

$$M_S = M_{S0} + M_{S1} \cdot X_1$$

Primer 5

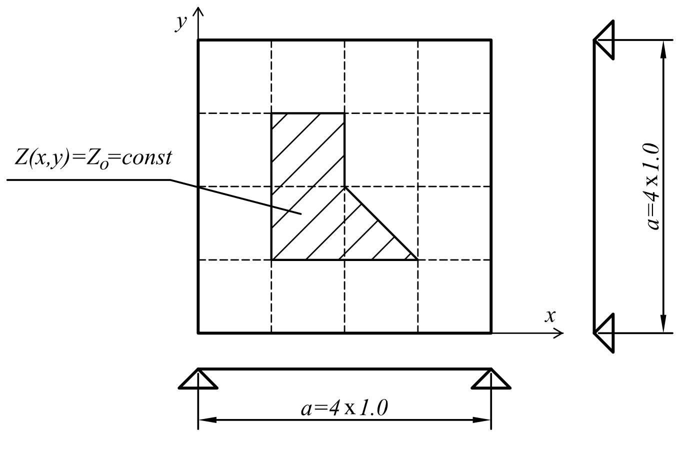
Za ploču prikazanu na slici odrediti ugib i momente savijanja u sredini ploče, koristeći samo prvi član reda usvojenog rešenja.

$Z(x, y) = Z_0 = 20\, kN/m^2$

$h = 0.15m$

$E = 30GPa$

$\nu = 0.15$



Rešenje:

$$w = A_{11}\sin\frac{\pi x}{a}\sin\frac{\pi y}{a}$$

$$A_{11} = \frac{Z_{11}}{K\pi^4\left(\frac{1}{a^2}+\frac{1}{a^2}\right)^2} = \frac{Z_{11}a^4}{4K\pi^4} \quad , \qquad Z_{11} = Z_{11}^{I} + Z_{11}^{II}$$

$$Z_{11}^{I} = \frac{4}{a^2}\int_1^2\int_1^3 Z_0\sin\frac{\pi x}{a}\sin\frac{\pi y}{a}dxdy = \frac{4Z_0}{a^2}\left(-\frac{a}{\pi}\right)\cos\frac{\pi x}{a}\Bigg|_1^2\left(-\frac{a}{\pi}\right)\cos\frac{\pi y}{a}\Bigg|_1^3 =$$

$$= \frac{4Z_0}{\pi^2}\left(\cos\frac{\pi}{2}-\cos\frac{\pi}{4}\right)\left(\cos\frac{3\pi}{4}-\cos\frac{\pi}{4}\right) = \frac{4Z_0}{\pi^2}$$

$$Z_{11}^{II} = \frac{4}{a^2}\int_2^3\int_1^{4-x} Z_0\sin\frac{\pi x}{a}\sin\frac{\pi y}{a}dxdy = \frac{4Z_0}{a^2}\int_2^3\left[\sin\frac{\pi x}{a}\int_1^{4-x}\sin\frac{\pi y}{a}dy\right]dx =$$

$$= -\frac{4Z_0}{a\pi}\left[\int_2^3\sin\frac{\pi x}{a}\cos\frac{\pi}{a}(4-x)dx - \frac{1}{\sqrt{2}}\int_2^3\sin\frac{\pi x}{a}dx\right] =$$

$$= -\frac{4Z_0}{a\pi}\left[\frac{1}{2}\int_2^3\left(\sin\frac{4\pi}{a}+\sin\frac{\pi}{a}(2x-4)\right)dx - \frac{1}{\sqrt{2}}\left(-\frac{a}{\pi}\right)\cos\frac{\pi x}{a}\Bigg|_2^3\right] =$$

$$= -\frac{4Z_0}{a\pi}\left[\frac{1}{2}\left(+\frac{a}{2\pi}\right)\int_0^{+\frac{2\pi}{a}}\sin t\,dt + \frac{a}{\sqrt{2}\,\pi}\left(\cos\frac{3\pi}{4}-\cos\frac{\pi}{2}\right)\right] = -\frac{4Z_0}{a\pi}\left(-\frac{a}{4\pi}\cos t\Bigg|_0^{+\frac{2\pi}{a}} - \frac{a}{\sqrt{2}\pi}\cdot\frac{1}{\sqrt{2}}\right) =$$

$$= -\frac{4Z_0}{a\pi} = \left(-\frac{a}{4\pi}\left(\cos\frac{\pi}{2}-1\right)-\frac{a}{2\pi}\right) = -\frac{4Z_0}{a\pi}\left(+\frac{a}{4\pi}-\frac{a}{2\pi}\right) = +\frac{4Z_0}{a\pi}\cdot\frac{a}{4\pi} = \frac{Z_0}{\pi^2}$$

$$\frac{\pi}{2}(2x-4) = t \qquad x = 2 \Rightarrow t = 0\ ,\ x = 3 \Rightarrow t = -\frac{2\pi}{a}$$

$$+\frac{2\pi}{a}dx = dt$$

$$\left[\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)\right]$$

$$Z_{11} = \frac{5Z_0}{\pi^2}$$

$$A_{11} = \frac{\frac{5Z_0}{\pi^2}\cdot a^4}{4K\pi^4} = \frac{5Z_0a^4}{4K\pi^6}$$

$$w(x,y) = \frac{5Z_0a^4}{4K\pi^6}\sin\frac{\pi x}{a}\sin\frac{\pi y}{a}$$

$$w\left(x = \frac{a}{2}; y = \frac{a}{2}\right) = \frac{5Z_0a^4}{4K\pi^6}$$

$$M_x = -K\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right)$$

$$T_x = -K\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) = ...0$$

$$M_{xy} = 0$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{5Z_0 a^2}{4K\pi^4}\sin\frac{\pi x}{a}\sin\frac{\pi y}{a} \qquad \left(\frac{\partial^2 w}{\partial x^2}\right)_{\substack{x=\frac{a}{2}\\ y=\frac{a}{2}}} = -\frac{5Z_0 a^2}{4K\pi^4}$$

$$\frac{\partial^2 w}{\partial y^2} = -\frac{5Z_0 a^2}{4K\pi^4}\sin\frac{\pi x}{a}\sin\frac{\pi y}{a} \qquad \left(\frac{\partial^2 w}{\partial y^2}\right)_{\substack{x=\frac{a}{2}\\ y=\frac{a}{2}}} = -\frac{5Z_0 a^2}{4K\pi^4}$$

$$M_x = -K\left(-\frac{5Z_0 a^2}{4K\pi^4}\right)\cdot(1+v) = \frac{5Z_0 a^2}{4\pi^4}(1+v)$$